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Entropy theory for derivation of infiltration equations

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[1] An entropy theory is formulated for modeling the potential rate of infiltration in unsaturated soils. The theory is composed of six parts: (1) Shannon entropy, (2) principle of maximum entropy (POME), (3) specification of information on infiltration in terms of constraints, (4) maximization of entropy in accordance with POME, (5) derivation of the probability distribution of infiltration, and (6) derivation of infiltration equations. The theory is illustrated with the derivation of six infiltration equations commonly used in hydrology, watershed management, and agricultural irrigation, including Horton, Kostiakov, Philip two-term, Green-Ampt, Overton, and Holtan equations, and the determination of the least biased probability distributions of these infiltration equations and their entropies. The theory leads to the expression of parameters of the derived infiltration equations in terms of measurable quantities (or information), called constraints, and in this sense these equations are rendered nonparametric. Furthermore, parameters of these infiltration equations can be expressed in terms of three measurable quantities: initial infiltration, steady infiltration, and soil moisture retention capacity. Using parameters so obtained, infiltration rates are computed using these six infiltration equations and are compared with field experimental observations reported in the hydrologic literature as well as the rates computed using parameters of these equations obtained by calibration. It is found that infiltration parameter values yielded by the entropy theory are good approximations.

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1. Introduction

[2] Infiltration is a key component of the hydrologic cycle. It partitions rainfall into surface runoff and that entering the soil. It is fundamental to determining the runoff hydrograph, soil moisture and groundwater recharge, irrigation efficiency, life span of pavements, and leaching of nutrients. Because of the fundamental role that infiltration plays in hydrology, irrigation engineering, and soil science, it has been a subject of much research for over a century. As a result a large number of infiltration equations have been developed, some of which are now commonly applied in hydrologic modeling and have been included in popular watershed hydrology models [*Singh*, 1989, 1995; *Singh and Frevert*, 2002a, 2002b, 2006; *Singh and Woolhiser*, 2002]. Some of the equations, commonly used in hydrology, watershed management, and agricultural irrigation, are *Horton* [1938], *Green and Ampt* [1911], *Kostiakov* [1932], Philip two-term [*Philip*, 1957], *Overton* [1964], and *Holtan* [1961] equations. Although these equations are simple and easy to use, one of the main difficulties is that their parameters need to be calibrated using field or experimental measurements [*Singh and Yu*, 1990]. These equations represent the potential rate of infiltration at a point under the condition that water supply is not a limiting factor. In order to compute the actual rate of infiltration under complex rainfall or water supply conditions, these equations are used in the same way that they are used for computing rainfall excess in hydrograph computations [*Bauer*, 1974; *Chu*, 1978; *Mls*, 1980; *Morel-Seytoux*, 1981; *Peschke and Kutilek*, 1982; *Singh*, 1989] where two conditions are satisfied. First, for a given rainfall event the amount of infiltration is computed using the mass balance (i.e., amount of infiltration equals amount of rainfall minus amount of runoff minus amount of abstractions). Second, this amount of infiltration is distributed over the duration of rainfall event using an infiltration equation, keeping in mind that the rate of infiltration during a given time interval will be less than or equal to the rainfall intensity. This computation is done iteratively. It may be noted that these infiltration equations are applicable to soil matrix, not macropores or soils with fractures.

[3] Soil characteristics, which govern the rate of infiltration, vary significantly from one place to another, and antecedent soil moisture, which defines the initial infiltration, also significantly varies spatially. The infiltration parameters determined using point measurements are point values, or at best reflect average values. Although large spatial variability in infiltration is recognized, little effort has been made to account for its probabilistic characteristics, except for a few watershed models, as for example, the BASINS (formerly Stanford Watershed Model) [*Crawford and Linsley*, 1966; *Donigian and Imhoff*, 2006].

[4] In recent years entropy has been applied to a range of problems in hydrology, and environmental and water

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resources engineering (see, for example, a review by *Singh* [1997]). A majority of applications have encompassed derivation of frequency distributions and estimation of their parameters, monitoring and evaluation of networks, and flow forecasting. More recently, *Koutsoyiannis* [2005a, 2005b] applied entropy to investigate the scaling behavior of hydrologic processes in both space and time, and emphasized the dominance of uncertainty in hydrologic processes. In a different vein, *Chiu* [1987, 1988, 1989, 1991] employed entropy to derive velocity distributions in open channel flow using only the mass conservation law. *Barbé et al.* [1991] extended Chiu's work by also including both momentum and energy conservation laws. Using both field and laboratory data, it was shown that the entropy-based velocity distributions were superior to the well-known power law and Prandtl-von Karman velocity distributions. More importantly, these investigators showed how entropy could be coupled with the laws of mathematical physics to derive useful results, such as velocity distributions in open channels and pipes, dispersion coefficients, and momentum and energy distribution coefficients. At a given time, the velocity distribution, as a function of flow depth, in a given open channel cross section is usually considered deterministic. However, the velocity distribution in the same cross section varies in time and therefore it is plausible to treat the time averaged velocity as a random variable, as *Chiu* [1987] did. This line of investigation provided the motivation to investigate if entropy theory could be developed for modeling infiltration.

[5] The objective of this study therefore is to propose an entropy theory for modeling infiltration into soils; use the theory to derive the well-known infiltration equations of Horton, Kostiakov, Philip, Green-Ampt, Overton, and Holtan; derive probability distributions of infiltration rate; define parameters of infiltration equations in terms of measurable information on infiltration known as constraints, and test the derived forms of these equations using experimental observations reported in the literature. The theory leads to the expression of infiltration equation parameters in terms of what is easily measured or measurable and hence to the physical basis of the parameters. The theory also establishes a probabilistic basis of infiltration equations and hence an estimate of uncertainty associated with each equation. The objective here is not to validate the equations or show if one equation is better than others or to investigate into their limitations and strengths.

2. Entropy Theory

[6] Let the rate of infiltration as a function of time t be defined as $I(t)$. It is assumed that the soil is dry and water is applied to the dry soil without limiting water supply. At the beginning, infiltration will be high and as time progresses, the rate of infiltration declines and may reach a steady or constant rate or approach zero. The rate of infiltration will be the potential or capacity rate. The objective is to derive the rate of infiltration as a function of time using the entropy theory. *Crawford and Linsley* [1966] were probably the first to consider spatial variations in infiltration capacity; from empirical data reported in the literature [*Burgy and Luthin*, 1956] they found large variations in infiltration capacity even in relatively homogeneous soils (uniform Yolo silt loam) and over small areas (40 feet by 20 feet). Considering infiltration capacity as a random variable they expressed the cumulative probability distribution of infiltration capacity as a function of area. Motivated by this work and recognizing that the infiltration rate may significantly vary from one place to another, it was assumed in this study that the spatially averaged rate of infiltration $I(t)$ is a random variable and would therefore have a probability density function. It is recognized that this assumption needs to be verified or may even be tenuous but even if it is weakly true it would not greatly mar the usefulness of the entropy theory. Let the probability density function of infiltration be defined as $f(I)$. Then, an entropy theory for infiltration capacity rate I can be formulated as comprising six parts: (1) Shannon entropy, (2) principle of maximum entropy (POME), (3) specification of information on infiltration in terms of constraints, (4) maximization of entropy in accordance with POME, (5) determination of the least biased probability density function of infiltration rate and maximum entropy, and (6) derivation of infiltration equations. Each of these parts is outlined in what follows.

2.1. Shannon Entropy

[7] Considering entropy as a measure of information and hence of uncertainty, *Shannon* [1948] formulated what is referred to as the Shannon entropy theory. The Shannon entropy quantitatively measures the mean uncertainty associated with a probability distribution of a random variable and in turn with the random variable itself in concert with several consistency requirements [*Kapur and Kesavan*, 1992]. For the probability density function of infiltration rate I, $f(I)$, the Shannon entropy, denoted by $H(I)$, can be expressed in general form [*Jaynes*, 1958, 2003] as

$$H(I) = -K \int_{I_L}^{I_U} f(I) \ln[f(I)/\varsigma(I)]dI \tag{1a}$$

where I_U and I_L are, the upper and lower limits of integration for I, respectively; $K > 0$ is an arbitrary constant or scale factor depending on the choice of measurement units; and $\varsigma(I)$ is an invariant measure function which guarantees the invariance of $H(I)$ under any allowable change of variable and provides an origin of measurement. The upper and lower limits of infiltration I_U and I_L may vary from one infiltration equation to another. For example, for the Horton infiltration equation the upper limit is defined by the initial infiltration I_0 and the lower limit by the steady infiltration rate I_c; whereas for the Kostiakov the upper limit is infinity and the lower limit is zero. What these limits will be for different infiltration equations will be clear from their derivations and are also given later in the text. The quantity $\varsigma(I)$ can also be interpreted as a prior distribution [*Jaynes*, 1958]. Scale factor K can be absorbed into the base of logarithm, and the invariant function $\varsigma(I)$ is usually taken as unity. Therefore, equation (1a) is often expressed [*Shannon and Weaver*, 1949, p. 87] as

$$H(I) = -\int_{I_L}^{I_U} f(I) \ln\, f(I)dI \tag{1b}$$

From equation (1b), H can be considered to describe the expected value of ($-\ln f(I)$). Considering ($-\ln f(I)$) as a measure of uncertainty or information gain, equation (1b) defines the average uncertainty associated with $f(I)$ and in turn with I. More uncertain I is, more information will be needed to characterize it. In other words, information reduces uncertainty. In this sense, uncertainty and information are related to each other. Thus, the key in equation (1b) is to derive the least biased $f(I)$.

2.2. POME

[8] The principle of maximum entropy formulated by *Jaynes* [1958, 1982] says that the least biased probability distribution of I, $f(I)$, will be the one that will maximize $H(I)$ given by equation (1b), subject to the given information on I expressed as constraints. In other words, if no information other than the given constraints is available then the probability distribution should be selected such that it is least biased toward what is not known. Such a probability distribution is yielded by the maximization of the Shannon entropy. Thus, one of the key points is to define constraints on I.

2.3. Constraints

[9] Information on $I(t)$ can be obtained using the knowledge of soil physics and experimental observations. For a given soil, one frequently measures cumulative infiltration and then characterizes the soil infiltration characteristics and more particularly the time rate of infiltration or infiltration curve under the condition that water is not a limiting factor for the soil. If infiltration rate observations are available, then one way to express information on the infiltration rate is in terms of constraints C_r, $r = 0, 1, 2, \ldots, n$, defined as

$$C_0 = \int_{I_L}^{I_U} f(I)dI = 1 \tag{2}$$

$$C_r = \int_{I_L}^{I_U} g_r(I)\ f(I)dI = \overline{g_r(I)}, \quad r = 1, 2, \ \ldots, n \tag{3}$$

where $g_r(I)$, $r = 1, 2, \ldots, n$, represent some functions of I, n denotes the number of constraints, and $\overline{g_r(I)}$ is the expectation of $g_r(I)$. In general, functions $g_r(I)$ for defining constraints can be simple functions. For example, if $r = 1$ and $g_1(I) = I$, equation (3) would correspond to the mean infiltration rate $\overline{I}$; likewise, for $r = 2$ and $g_2(I) = (I - \overline{I})^2$, it would denote the variance of I. The choice of functions $g_r(I)$ depends on the knowledge of infiltration physics, the ease with which they can be specified, the simplicity of the ensuing algebra, and the availability of observations. For most infiltration equations used in hydrology, more than two constraints are not needed.

2.4. Maximization of Shannon Entropy

[10] In order to obtain the least biased $f(I)$, the entropy given by equation (1b) is maximized, subject to equations (2) and (3), and one simple way to achieve the maximization is the use of the method of Lagrange multipliers. To that end, the Lagrangian function L can be constructed as

$$L = -\int_{I_L}^{I_U} f(I)\ln f(I)dI - (\lambda_0 - 1)\left[\int_{I_L}^{I_U} f(I)dI - C_0\right] - \sum_{r=1}^{n} \lambda_r \left[\int_{I_L}^{I_U} f(I)g_r(I)dI - C_r\right] \tag{4}$$

where $\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_n$ are Lagrange multipliers. In order to obtain $f(x)$ which maximizes L, one may recall the Euler-Lagrange equation of the calculus of variations, and therefore one differentiates L with respect to $f(I)$ (noting I as parameter and f as variable) and equates the derivative to zero and obtains

$$\frac{\partial L}{\partial f} = 0 \Rightarrow -[1 + \ln f(I)] - (\lambda_0 - 1) - \sum_{r=1}^{n} \lambda_r g_r(I) = 0 \tag{5}$$

2.5. Probability Distribution of Infiltration Rate

[11] Equation (5) leads to the probability density function of I in terms of the given constraints:

$$f(I) = \exp\left[-\lambda_0 - \sum_{r=1}^{n} \lambda_r g_r(I)\right] \tag{6a}$$

where the Lagrange multipliers λ_r, $r = 0, 1, 2, \ldots, n$, can be determined with the use of equations (2) and (3). Integration of equation (6a) leads to the cumulative distribution function or simply probability distribution of I, $F(I)$:

$$F(I) = \int_0^I \exp\left[-\lambda_0 - \sum_{r=1}^{n} \lambda_r g_r(x)\right] dx \tag{6b}$$

[12] Substitution of equation (6a) in equation (1b) results in the maximum entropy of $f(I)$ or I:

$$H = -\lambda_0 - \sum_{r=1}^{n} \lambda_r \overline{g_r(I)} \tag{7}$$

Equation (7) shows that the entropy of the probability distribution of infiltration rate or of the rate of infiltration itself depends only on the constraints, since the Lagrange multipliers themselves can be expressed in terms of the specified constraints. Equations (1b) to (3), (6a), and (7) constitute the building blocks of the entropy theory which is now illustrated by deriving a general infiltration equation.

2.6. Derivation of Infiltration Equations

[13] Consider a soil element, without macropores or fractures, receiving water. It is assumed that the soil is dry and there is no limit to water supply. As water infiltrates, the pore spaces begin to get filled up, and the potential for infiltration begins to decline, i.e., the infiltration capacity rate

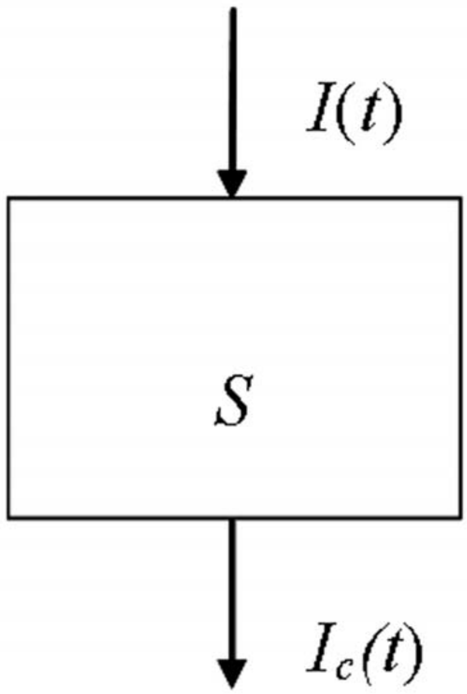

Figure 1. Soil element.

as a function of time t, $I(t)$, starts to decline. Let the rate at which water will be exiting the soil element be denoted as I_c which approximately equals the steady state or constant rate of infiltration. The soil will have a maximum soil moisture retention capacity, denoted by S. For a dry soil, S will be equal to the soil porosity multiplied by the soil elemental volume minus the volume of pore spaces which are not occupied by roots, earthworms, or other objects. The maximum water retained will be the same as cumulative infiltration (denoted as J), that is, $0 \le J \le S$. The continuity equation for the soil element, as shown in Figure 1, can be expressed [*Singh and Yu*, 1990] as

$$\frac{dJ(t)}{dt} = I(t) - I_c; \quad or \quad J(t) = \int_0^t I(x)dx - I_c t \tag{8}$$

where t is time. If W is the amount of pore space available for infiltration of water at any time, then $W + J = S$.

[14] It is hypothesized that the cumulative probability distribution function (CDF) of infiltration $F(I)$ can be defined as the ratio of soil moisture potential (W) to the maximum soil moisture retention (S):

$$F(I) = \frac{W}{S} \tag{9a}$$

$F(I)$ can also be defined as one minus the ratio of cumulative infiltration J to the maximum soil moisture retention, S (or maximum or potential cumulative infiltration):

$$F(I) = 1 - \frac{J}{S} \tag{9b}$$

It may be noted that the argument of F on the left side is I, not J. The hypothesis expressed by equation (9b), however, needs to be validated using empirical measurements. Differentiation of equation (9b) yields

$$dF(I)dI = -\frac{dJ}{S}; \qquad dF(I) = f(I) = -\frac{1}{S}\frac{dJ}{dI} \tag{9c}$$

where $f(I)$ is the probability density function (PDF) of I which is determined using the entropy theory.

[15] Substitution of equation (6a) in equation (9c) yields

$$\exp\left[-\lambda_0 - \sum_{r=1}^{n} \lambda_r g_r(I)\right] dI = -\frac{1}{S} dJ \tag{10a}$$

Integrating equation (10a) with limits on J from 0 to J and on I from I_U to I, one obtains

$$J = S \int_I^{I_U} \exp\left[-\lambda_0 - \sum_{r=1}^{n} \lambda_r g_r(x)\right] dx \tag{10b}$$

Equation (10b) is a general relation between I and J, and can be integrated, depending on the form of $g_r(I)$, $r = 1, 2, \ldots, n$. Upon integration, I can be expressed as $I(t) - I_c = dJ(t)/dt$ and hence $J(t)$ can be determined. Then, differentiating $J(t)$ with respect to t will lead to an expression for $I(t)$ which is what is desired. This procedure or application of the entropy theory is now illustrated by deriving six popular infiltration equations, including the Horton, Kostiakov, Philip, Green-Ampt, Overton, and Holtan equations. For illustration, the Horton equation is derived here, and other infiltration equations are derived in Appendices A–E.

2.7. Horton Equation

[16] For the Horton equation, the beginning infiltration rate is defined as I_0. As time progresses, the rate of infiltration declines and reaches a steady or constant rate denoted as I_c. The objective is to derive the rate of infiltration as a function of time. For simplicity, let the excess infiltration rate be defined as $i(t) = I(t) - I_c$, and $i_0 = I_0 - I_c$. Thus, $i(t)$ will vary from 0 to i_0, whereas $I(t)$ will vary from I_c to I_0.

[17] Let the probability density function of i be denoted as $f(i)$. The simplest constraint that $f(i)$ must satisfy is

$$C_0 = \int_0^{i_0} f(i)di = 1 \tag{11}$$

Applying POME and using the method of Lagrange multipliers [*Singh*, 1998], one obtains

$$f(i) = \exp(-\lambda_0) \tag{12a}$$

where λ_0 is the zeroth Lagrange multiplier.

[18] Inserting equation (12a) in equation (11), one gets

$$\int_0^{i_0} \exp(-\lambda_0)di = 1 \Rightarrow f(i) = \frac{1}{i_0} \qquad or \qquad f(I) = \frac{1}{I_0 - I_c} \tag{12b}$$

Equation (12b) is the probability density function of the Horton equation uncovered by the entropy theory. It states that the probability density function of i or I is a uniform distribution which may not be realistic, for smaller values of infiltration close to I_c are more likely to occur, leading to a skewed PDF. In field experiments, Horton equation seems to simulate infiltration reasonably well, despite its uniform PDF. This may suggest relative insensitivity of the Horton infiltration rate to the underlying PDF. For a given soil I_c is usually constant, but I_0 depends on the antecedent moisture condition and may vary in time. This means that probability density function $f(I)$ will also vary in time with time varying values of I_0 and I_c.

[19] From equations (12a) and (12b), one obtains the Lagrange multiplier λ_0 as

$$\lambda_0 = \ln i_0 = \ln(I_0 - I_c) \tag{13}$$

The cumulative distribution function of i or I would be linear, expressed by integration of equation (12b) as

$$F(i) = \int_0^i f(x)dx = \int_0^i \frac{1}{i_0}dx = \frac{i}{i_0}, \quad or \quad F(I) = \frac{I - I_c}{I_0 - I_c} \tag{14}$$

[20] Combining equations (9c) and (12b), one obtains

$$\frac{1}{i_0}di = -\frac{1}{S}dJ \tag{15a}$$

Integrating equation (15a), one obtains

$$\frac{1}{i_0}(i - i_0) = -\frac{J}{S} \quad or \quad i = i_0 - \frac{i_0 J}{S} \tag{15b}$$

Equation (15b) can be recast as

$$\frac{dJ}{dt} = i_0 - \frac{i_0}{S}J \tag{16a}$$

Solution of equation (16a) yields the cumulative infiltration as

$$J = S\left[1 - \exp\left(-\frac{i_0}{S}t\right)\right] \tag{16b}$$

Differentiating equation (16b) with respect to t and recalling the continuity equation (8) one obtains the excess infiltration rate as

$$i(t) = i_0 \exp\left(-\frac{i_0}{S}t\right) \tag{17}$$

Since I_c was subtracted from $I(t)$ as well as I_0, equation (17) is now written in original terms as

$$I(t) = I_c + (I_0 - I_c)\exp\left(-\frac{(I_0 - I_c)}{S}t\right) \tag{18}$$

Equation (18) can be written as

$$I(t) = I_c + (I_0 - I_c)\exp(-t/k) \tag{19}$$

where

$$k = \frac{S}{(I_0 - I_c)} \tag{20}$$

Equation (19) is the Horton infiltration equation derived using the entropy theory. Another interesting aspect here is that the theory also couples the mass balance equation.

[21] The entropy of the probability distribution of the Horton equation or of the infiltration rate can be obtained by substituting equation (12b) in equation (1):

$$H(i) = -\int_0^{i_0} \frac{1}{i_0}\ln\frac{1}{i_0}\,di = \ln i_0 \quad \text{or} \quad H(I) = \ln(I_0 - I_c) \tag{21a}$$

Equation (21) states that the uncertainty of $f(I)$ or I for that matter depends on the initial value of I, I_0, and the steady state value of I, I_c. It may be noted that in light of equation (1a), the entropy of the Horton equation should be written as

$$H(I) = \ln[\varsigma(I_0 - I_c)] \tag{21b}$$

where measure ς will have the units inverse of those of I and thus it would allow the units of H in the usual entropy terms (Napiers or bits). In equation (21a) ς was taken as one but its units were retained, making the argument of the logarithm dimensionless. An important implication of equation (21a) is that for a given soil the uncertainty of the Horton equation is maximum when it is dry, because that is when the initial infiltration will be maximum and reduces as soil becomes wetter. This means that when sampling infiltration, greater care should be exercised in the beginning of infiltration and less toward the tail. This also means that infiltration observations should be more closely spaced temporally in the beginning but the time interval between observations can be increased with the progress of infiltration.

[22] Derivation of equation (19) shows that the Horton equation requires no constraint other than the total probability theorem which is not a constraint in a true sense, for all probability distributions must satisfy it. In equation (19) parameter k is expressed as the ratio of the maximum soil moisture retention and the initial infiltration rate minus the steady state infiltration rate. Parameter k has the dimension of time and indicates the time it will take for the infiltrated water to fill the maximum soil moisture retention space, if the rate of infiltration were the initial infiltration rate (i.e., the maximum infiltration rate) minus the steady rate, or the initial excess infiltration rate. One can interpret $(I_0 - I_c)$ as the average velocity of water at which pore space S is filled. Infiltration observations provide initial and steady infiltration rates and for a given a soil with the knowledge of its porosity and its column height the value of S (the maximum soil moisture retention) can be obtained. Thus, parameter k can be computed using equation (20) without any calibration. This also provides a physical interpretation of parameter k.

[23] In the event that infiltration observations are not available, the maximum soil moisture retention parameter S can be obtained from the SCS-CN (Soil Conservation Service

Curve Number) method [*Soil Conservation Service*, 1972; *Mishra and Singh*, 2003] as

$$S = \frac{1000}{CN} - 10 \tag{22}$$

where CN is the curve number derived from the knowledge of antecedent soil moisture, soil type, land use and hydrologic condition. The values of CN are extensively tabulated [*Mishra and Singh*, 2003]. In this manner the entropy theory reinterprets the Horton equation in a useful way.

3. Derivation of Other Infiltration Equations

3.1. Kostiakov Equation

[24] The Kostiakov equation [*Kostiakov*, 1932] is derived in Appendix A and can be expressed as

$$I(t) = 0.5at^{-0.5} \tag{23}$$

where a is parameter. From the entropy theory, one obtains $a = (2I_cS)^{0.5}$, twice the product of steady infiltration (I_c) and maximum soil moisture retention (S) both of which can be determined for a given soil. This means that parameter a can be obtained from observations and does not need to be calibrated.

3.2. Philip Two-Term Equation

[25] The Philip two-term equation [*Philip*, 1957] is derived in Appendix B and can be expressed as

$$I(t) = a_0 + 0.5(2cS)^{0.5}t^{-0.5} = a_0 + bt^{-0.5},$$
$$b = 0.5(2cS)^{0.5}, \quad c \approx a_0 \tag{24}$$

where a_0 and b are parameters. Parameter a_0 is analogous to steady infiltration rate (or saturated hydraulic conductivity) and can be obtained without having any calibration. In general, a_0 is between 0.5 and 0.75 of the saturated hydraulic conductivity (or I_c), and parameter c is approximately taken as equal to a_0. Parameter b can be expressed in terms of a_0 and maximum soil moisture retention S which also can be obtained from observations. Thus, parameters a_0 and b have physical meaning and need no calibration. Equation (24) shows that there is an interaction between parameters b and a_0.

3.3. Green-Ampt (G-A) Equation

[26] The G-A equation [*Green and Ampt*, 1911] is derived in Appendix C and can be expressed as

$$t = \frac{1}{I_c}\left[J - \frac{a_1}{I_c}\log\left(1 + \frac{J}{a_1/I_c}\right)\right], \qquad a_1 = SI_c \tag{25}$$

where a_1 is parameter. In equation (25) parameter I_c is the steady state rate of infiltration and can be interpreted as almost equal to the saturated hydraulic conductivity. Parameter S is the maximum soil moisture retention. Since I_c and S can be obtained from observations, $a_1 = SI_c$ can also be obtained from observations. In the hydrologic literature, S is interpreted as equal to the product of the capillary suction at the wetting front and the initial moisture deficit. The entropy theory provides another interpretation of parameter S and hence the G-A parameters can be estimated without calibration.

3.4. Overton Equation

[27] The Overton equation [*Overton*, 1964] is derived in Appendix D and can be written as

$$I(t) = I_c \sec^2\left[\sqrt{a_2 I_c}(t_c - t)\right], \qquad a_2 = \frac{S^2}{(I_0 - I_c)} \tag{26}$$

where a_2 is parameter, and t_c is the time to steady state infiltration I_c; this time may be much smaller than the duration of the infiltration experiment or observations and can be obtained from observations. Since a_2 is expressed as $(I_0 - I_c) = a_2S^2$ in which I_0 is the initial infiltration, parameters of the Overton equation can be obtained from observations and calibration of these parameters may not be needed.

3.5. Holtan Equation

[28] The Holtan equation [*Holtan*, 1961] is derived in Appendix E and can be expressed as

$$I = I_c + a_3\left[S^{1-m} - (1-m)a_3t\right)^{\frac{m}{1-m}} \tag{27}$$

where a_3 is a parameter expressed as

$$a_3 = \frac{(I_0 - I_c)}{S^m} \tag{28}$$

and

$$m = \ln(I_0 - I_c) - \overline{\ln(I - I_c)} \tag{29}$$

Parameters a_3 and m can be obtained from observations as equations (28) and (29) show and calibration may therefore not be needed.

4. Testing

4.1. Infiltration Data

[29] Data on infiltration in field soils have been reported by *Rawls et al.* [1976] in a report published by the Agriculture Research Service of the U.S. Department of Agriculture. Four datasets (labeled as I, II, III, IV) on infiltration in Robertsdale loamy sand, Stilson loamy sand, and Troupe sand in the Georgia Coastal Plain were obtained and used in this study for purposes of illustrating the application and usefulness of the entropy theory. Characteristics of infiltration observations are given in Table 1. In Table 1, D is the duration of the experiment in minutes; t_c is the time to the approximately constant rate of infiltration in minutes, and it may be less than the duration of the experiment D; I_c is the constant (steady) rate of infiltration (cm/h) at the end of infiltration experiment or the duration D applied to all the equations except for the Overton equation; I_c' is the constant rate of infiltration (cm/h) at time $t = t_c$ (which occurs before the end of the experiment) and is applied to the Overton

Table 1. Parameters From Observations After *Rawls et al.* [1976][a]

Soil Type	Code	Identification Number	I_0 (cm/h)	I_c (cm/h)	I_c' (cm/h)	S_1 (cm)	S_2 (cm)	S_3 (cm)	S' (cm)	t_c (min)	Duration of Observations, D (min)
Robertsdale Loamy Sand	I	09091D	12.21	2.42	3.10	4.17	7.61	4.28	2.77	50	120
Robertsdale Loamy Sand	II	09091W	8.24	2.25	1.93	0.76	4.90	2.40	0.40	50	120
Stilson Loamy Sand	III	10101W	12.81	2.97	2.96	1.68	7.04	4.99	2.54	50	91
Troup Sand	IV	12112W	11.60	4.40	4.37	2.59	12.14	11.21	3.12	110	123

[a]For data sets I and II, rainfall was applied at a uniform rate of 12.21 cm/h (4.807 in./h) until a constant runoff rate was achieved. It was stopped for 60 min and then applied again at a lower rate until constant runoff rate was achieved. The total duration for rainfall application was 120 min. For data set III, the initial rate was 12.82 cm/h (5.047 in./h), and the duration was 91 min. For data set IV, the initial rainfall rate was 16.51 cm/h (6.5 in./h), and the duration was 112 min.

equation; I_0 is the initial infiltration rate (cm/h) given 4 min later than the start of infiltration ($t = 0$) applied to all the six equations. The initial time of observation was 4 min for data sets I and II, 8 min for data set III and 5 min for data set IV.

[30] For the dataset I, the infiltration rate reached a lower value at $t = 50$ min and thereafter fluctuated round 3.10 cm/h (1.22 in./h) (corresponding to the cumulative infiltration of 4.28 cm (1.686 in.)). Thus in this case $t_c = 50$ min and $I_c' = 3.10$ cm/h (1.22 in./h). The initial infiltration rate at $t = 4$min was 12.21 cm/h (4.807in./h). The actual initial infiltration rate (at $t = 0$) should be larger than 12.21 (cm/h) which is the value at $t = 4$ min. For computation, the value of I_0 used was the value observed at $t = 4$ min. It is recognized that this is not the correct value but no observations at $t = 0$ were available. It was assumed that the infiltration rate at the end of the experiment reached the constant infiltration rate and it was therefore assumed that the constant infiltration rate I_c was 2.42 cm/h (0.953 in./h) which is the value of the infiltration rate at the end of the experiment. Since the connotation of parameter S may differ from one infiltration equation to another, it may have different values for different equations. Therefore, S_1 was used to denote parameter S for the Green-Ampt equation as the maximum soil moisture retention determined by subtracting the final soil moisture content minus the initial soil moisture content, while S_2 was used to denote the cumulative infiltration until time D applied to the Kostiakov and Philip equations; likewise, S_3 for the Overton equation was used to denote parameter S equal to the accumulated infiltration until t_c. S' was used to denote parameter S for the Horton model, which was determined as $S' = S_2 - I_c \times D$; these parameter values were obtained from observations and are given in Table 1. In a similar manner, values of I_c, I_c', I_0, S_1, S_2, S_3, and t_c were obtained for data sets II, III, and IV, and are shown in Table 1.

4.2. Validation of Infiltration Hypothesis

[31] Equation (9b) is a hypothesis fundamental to deriving the aforementioned infiltration equations or may be even other equations. This hypothesis was tested for the above

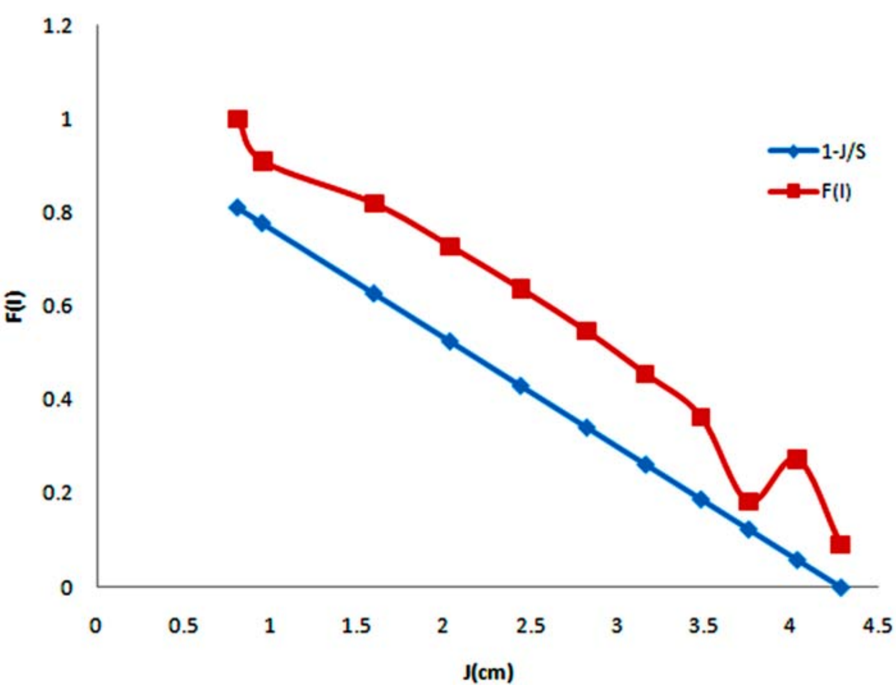

Figure 2. Cumulative distribution function of infiltration capacity rate as a function of cumulative infiltration for data set I.

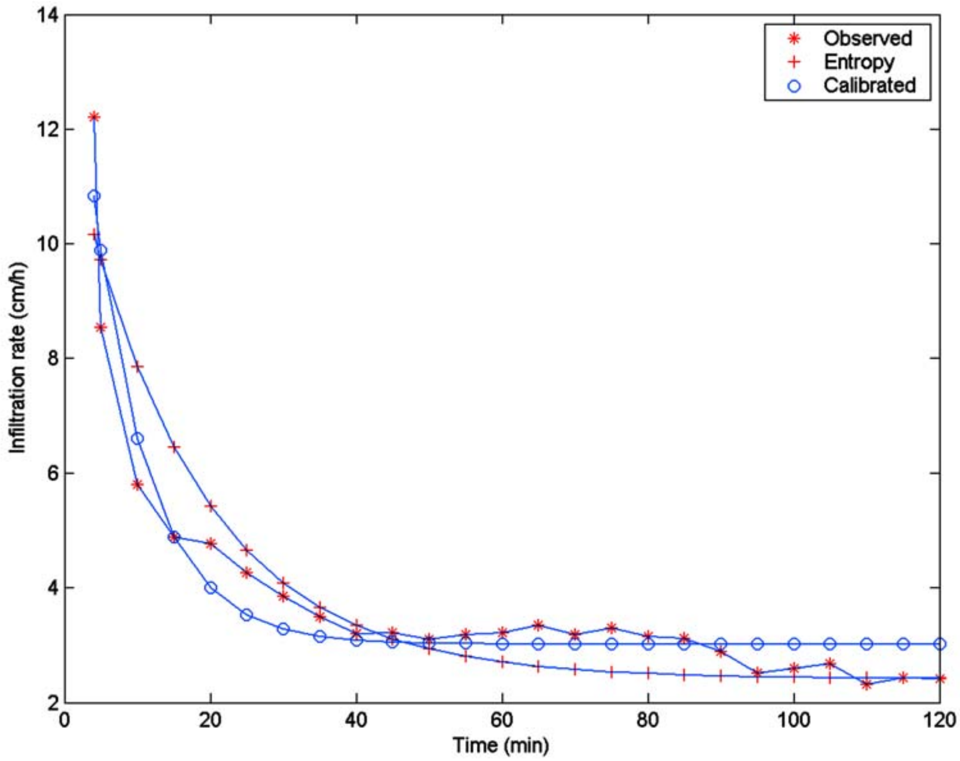

Figure 3. Comparison of infiltration rates computed using the Horton equation with parameters determined using entropy theory and by calibration with observed infiltration rates for data set I.

four data sets, and for a sample data set I, it is shown in Figure 2. The results for the other three data sets were similar. The field data plotted approximately as a straight line, and it may be argued that the hypothesis is approximately valid, but needs to be tested much more extensively. It may however be emphasized that the less than perfect validity of this hypothesis does not diminish the usefulness of the entropy theory.

4.3. Horton Equation

[32] The Horton equation has three parameters I_c, I_0, and k as shown in equation (19). In the usual hydrologic practice, these parameters are obtained by calibration or fitting the Horton equation to infiltration observations. In the case of the entropy theory parameters I_c and I_0 were obtained from observations and the value of S was also obtained from observations as explained earlier. Using these observed values of I_c, I_0, and S, parameter k was computed using equation (20). Thus, no calibration or fitting was done to obtain parameters I_c, I_0, and k. It may be noted that any error in data would directly translate into errors in the computed infiltration rates. On the other hand, these three parameters were also obtained by calibration using the least square method in which the sum of squares of deviations between observed and computed infiltration rates was minimized. This was for purposes of comparing the entropy theory-based infiltration rates with the infiltration rates obtained using calibrated parameter values.

[33] With parameter values obtained from observations using the entropy theory and from calibration, the Horton equation was applied to all four data sets. The infiltration rates computed in the above two ways and observed rates are shown in Figure 3. The infiltration rates computed using the entropy theory and calibration were in reasonable agreement with observed infiltration rates. Clearly, the infiltration rates obtained using the calibrated parameter values were in closer agreement with observed values. The average relative error (defined as the absolute difference between observed and computed rates divided by the observed rate) was under 13% for both the entropy theory and calibration. As expected, computed rates improved as time progressed. For other data sets II, III, and IV, the absolute average relative error was 10%, 9.8%, and 3.9%, respectively, for the entropy theory and under 6.9%, 2.6%, and 4.6% for calibration. On average, the entropy theory performed remarkably well for all data sets, especially when there was no adjustment of parameters. It was observed that for data sets I and III, the maximum relative error (at a certain point in time) was significantly higher for the entropy theory than for calibration. However, two points need to be noted. First, for the most part the relative error for the entropy theory was significantly lower, thus the error was not as high as the maximum value of the error would lead one to infer. Second, a closer examination of data set I revealed that the infiltration rate started to fluctuate at t = 50 min all the way up to the end of the experiment, D = 120 min. This was also the case for data set III where the infiltration rate started to fluctuate at t = 50 min. It was not clear what the reason for fluctuating infiltration rates was. It might be small macropores or experimental errors.

[34] Also computed was error equal to the square root of the mean of square of differences between computed and observed infiltration rates. Using the entropy based parameters, this error was 0.80, 0.83, 1.22, and 0.37 cm./h for data sets I, II, III, and IV, respectively. For the calibrated

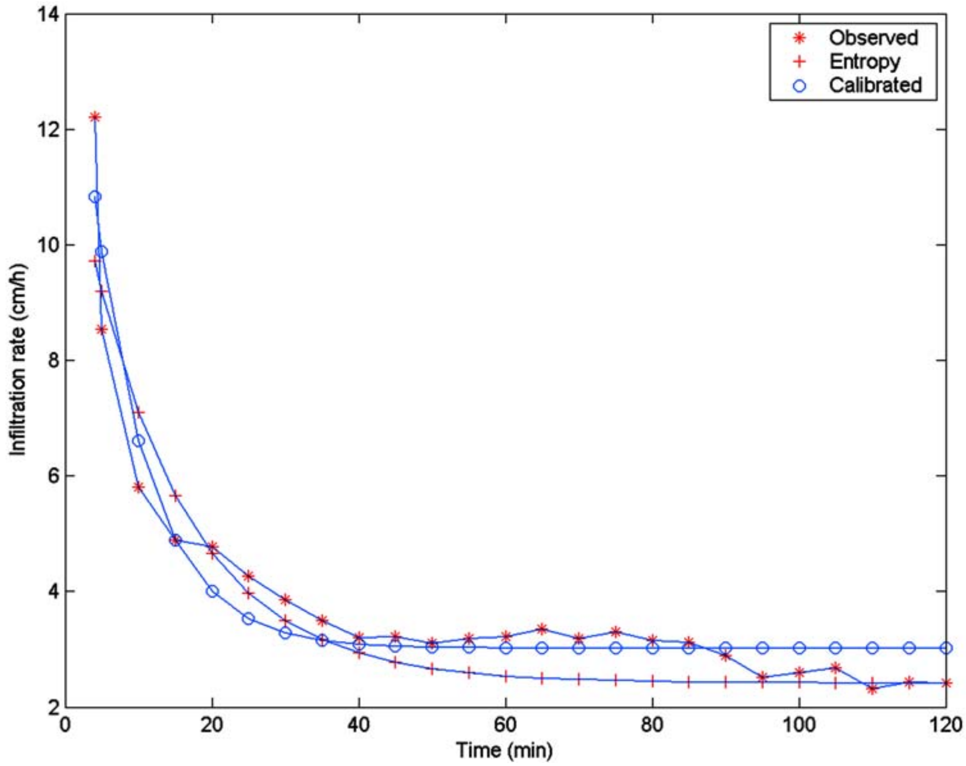

Figure 4. Sensitivity of the Horton equation to parameter S with S as 0.8 the original S for data set I.

parameters, the error was 0.57, 0.19, 0.16, and 0.35 cm/h. As expected, calibration produced infiltration rates closer to observed rates. Nevertheless, the entropy theory performed reasonably well. From now onward this error will be referred to as mean error.

[35] Furthermore, the available value of parameter S may not be accurate and hence a little bit of adjustment of the S value might lead to improved infiltration rates for the entropy theory. Therefore, parameter S was changed by plus or minus 10% to 40% with an increment of 10% in order to evaluate the sensitivity of infiltration rates to parameter S. Figure 4 shows infiltration rates when S was reduced by 20% or $S = 0.8S_0$ (S_0 was the value from observations). The infiltration rate values computed by the entropy theory improved, indicating that more accurate observations would lead to improved infiltration rate estimates by the entropy theory.

4.4. Kostiakov Equation

[36] This equation has only one parameter a which was obtained by calibration as well as directly from observations using equation (A17) due to the entropy theory. Figure 5 compares observed infiltration rates and the rates computed using the entropy theory and calibration for data set I. The computed rates in both cases were higher than the observed rates for time equal to about 57 min. The absolute average relative error was 12.23%, 35%, 14.4%, and 13% for data sets I, II, III, and IV, respectively, for the entropy theory, and 10%, 24.23%, 18.23%, and 3% for calibration. For the entropy based parameter, the mean error was 0.67, 1.17, 1.29, and 1.61 cm./h for data sets I, II, III, and IV, respectively. Using the calibrated parameter, the error was 0.56, 0.75, 0.75, and 0.20 cm/h. Clearly, the calibrated infiltration rates were closer to observed rates, although the entropy theory-based infiltration rates were reasonable. It may be noted that the value of parameter a as estimated for the entropy theory may be less than accurate, for the value of S as given in the data does not match the accumulated infiltration. Reducing this value of S would lead to improved infiltration rates owing to the entropy theory.

4.5. Philip Two-Term Equation

[37] The Philip equation has two parameters a_0 and I_c as shown in equation (B15). These parameters were estimated by calibration and from observations using equation (B15) for the entropy theory. Figure 6 compares observed infiltration rates and the rates computed using the entropy theory and calibration for data set I. Figure 6 shows that both methods overestimate infiltration up to about 58 min. The absolute average relative error was 9.6%, 35%, 4.7%, and 13% for data sets I, II, III, and IV for the entropy theory, and 8.5%, 21.33%, 20.6%, and 3.73% for calibration. For the entropy case, the mean error was 0.64, 1.05, 1.67, and 1.01 cm./h for data sets I, II, III, and IV, respectively. For calibration, the error was 0.53, 0.70, 0.92, and 0.24 cm/h. As expected, the mean error was less for calibration than for the entropy theory. Considering that there was no calibration for the entropy theory, it compared reasonably well with calibration. Reducing the value of S and/or a_0 would lead to improved infiltration rate estimates.

4.6. Green-Ampt Equation

[38] The G-A equation has two parameters a_1 and I_c as shown in equation (C15). These parameters were estimated by calibration and from observations using equation (C16)

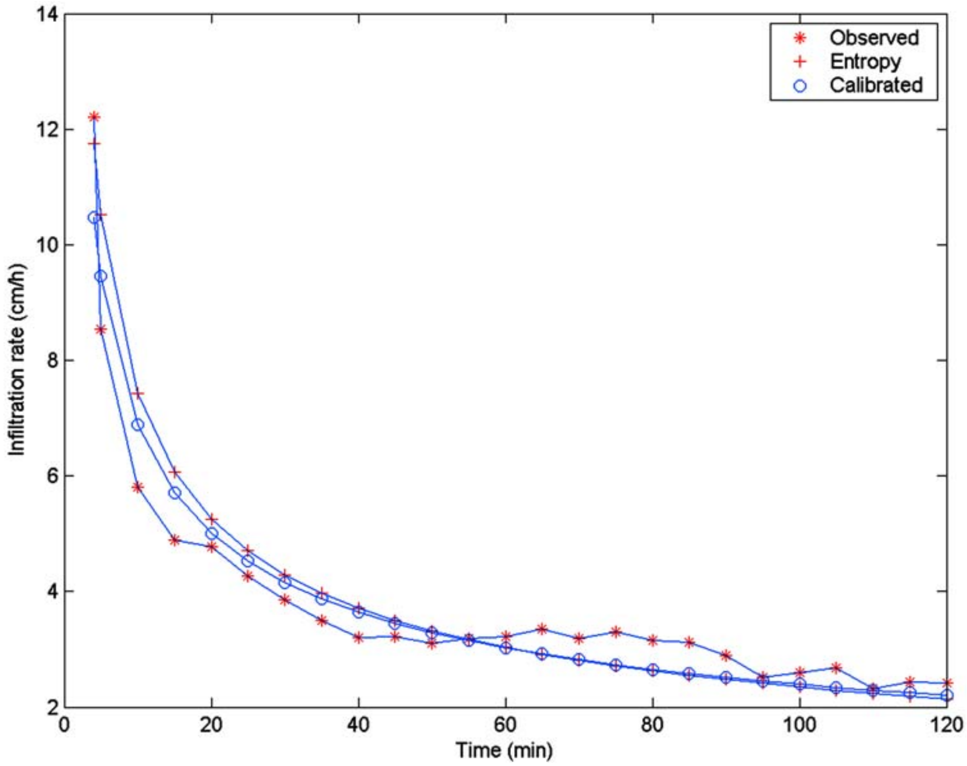

Figure 5. Comparison of infiltration rates computed using the Kostiakov with parameters determined using entropy theory and by calibration with observed infiltration rates for data set I.

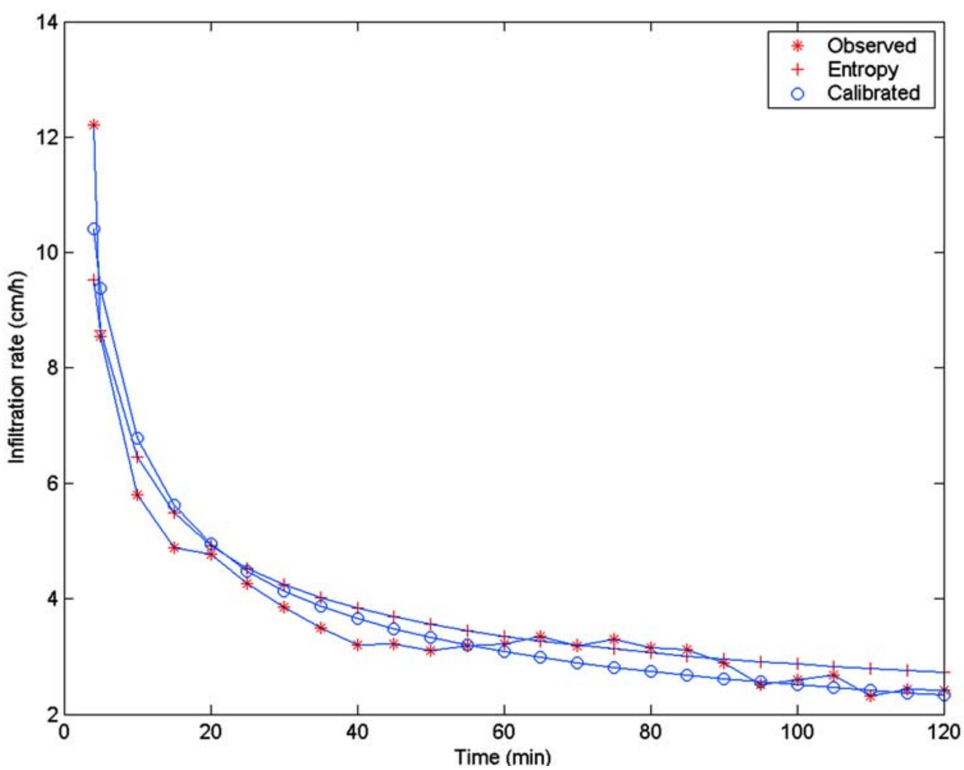

Figure 6. Comparison of infiltration rates computed using the Philip two-term equation with parameters determined using entropy theory and by calibration with observed infiltration rates for data set I.

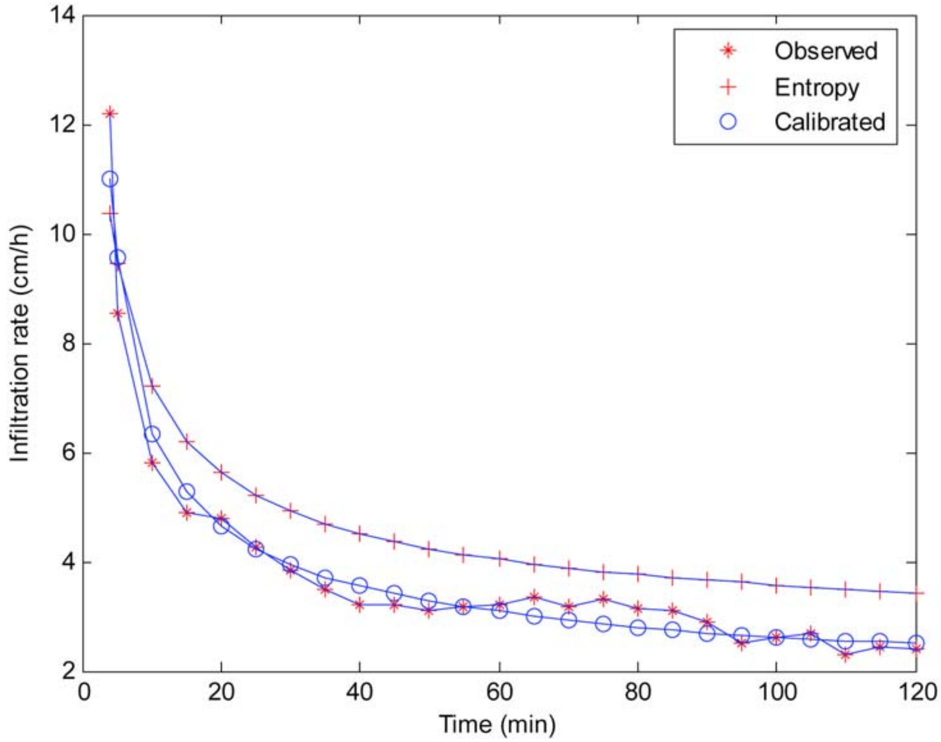

Figure 7. Comparison of infiltration rates computed using the Green-Ampt equation with parameters determined using entropy theory and by calibration with observed infiltration rates for data set I.

for the entropy theory. Figure 7 compares observed infiltration rates and the rates computed using the entropy theory and calibration for data set I. Figure 7 shows that the entropy theory consistently overestimates and the calibration method overestimates infiltration up to about 55 min and then it underestimates. The absolute average relative error for data sets I, II, III, and IV was 28.6%, 29.1%, 25.4%, and 10.9% for the entropy theory, and 6.6%, 22.1%, 17.6%, and 8% for calibration. For entropy-based infiltration rate, the mean error was 1.03, 0.93, 2.06, and 0.64 cm./h, for data sets I, II, III, and IV, respectively. For calibration, the mean error was 0.40, 0.69, 0.75, and 0.45 cm/h. In this case the entropy theory did not perform as well as it did for other equations. However, considering that there was no calibration of parameters, the performance was within error bounds that can be reduced. It was noticed that reducing the value of a_1 through S and I_c would lead to improved infiltration estimates.

4.7. Overton Equation

[39] The Overton equation has actually three parameters a_2, I_c and t_c as shown in equation (D16). These parameters were estimated by calibration and from observations using equation (D15) for the entropy theory. Figure 8 compares observed infiltration rates and the rates computed using the entropy theory and calibration for data set I. Figure 8 shows that the both entropy theory and the calibration method overestimated infiltration rate up to t = 18 min and then underestimated. The absolute average error for data sets I, II, III, and IV was 6.5%, 36.2%, 20.5%, and 4.64% for the entropy theory, and below 11.2%, 21.8%, 5.2%, and 4.35% for calibration. For the entropy theory, the mean error was 0.98, 1.70, 3.10, and 0.45 cm/h, for data sets I, II, III, and IV, respectively. For calibration, the error was 0.82, 0.73, 0.29, and 0.43 cm/h. Considering that there was no calibration for the entropy theory, it compared reasonably well with calibration. Reducing the value of a_2 through S and I_c would lead to improved infiltration estimates.

4.8. Holtan Equation

[40] The Holtan equation has three parameters a_3, I_c and m, as shown in equation (E17). These parameters were estimated by calibration and from observations using equation (E17) for the entropy theory. Their values were: I_c = 2.42 cm/h, a_3 = 0.93, and m = 2.3, by entropy and I_c = 2.82, a_3 = 3.14, and m = 2.3 by calibration, where m = 1.5 was fixed for both the methods. Figure 9 compares observed infiltration rates and the rates computed using the entropy theory and calibration for data set I. Figure 9 shows that the entropy theory overestimates infiltration a little bit, whereas the calibration method overestimates and underestimates. The absolute relative error for data sets I, II, III, and IV was 8.3%, 10.9%, 13.2%, and 7.5% for the entropy theory, and 8.8%, 6.3%, 5.2%, and 3.9% for calibration. Using the entropy based parameters, the mean error was 0.67, 0.94, 1.52, and 0.48 cm./h for data sets I, II, III, and IV, respectively. For the calibrated parameters, the error was 0.48, 0.16 0.24, and 0.29 cm/h. In this case the entropy theory yielded not as good estimates as did calibration. However, considering that there was no calibration of parameters, the theory performed remarkably well. Reducing the value of a_3 through S and I_c would lead to improved infiltration estimates.

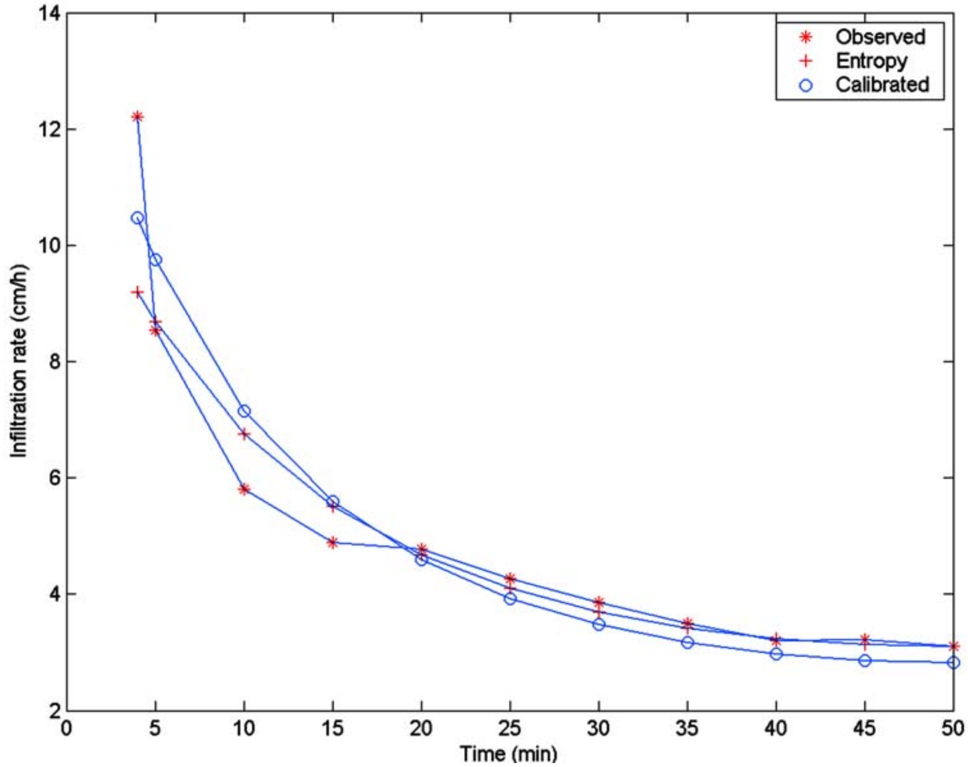

Figure 8. Comparison of infiltration rates computed using the Overton equation with parameters determined using entropy theory and by calibration with observed infiltration rates for data set I.

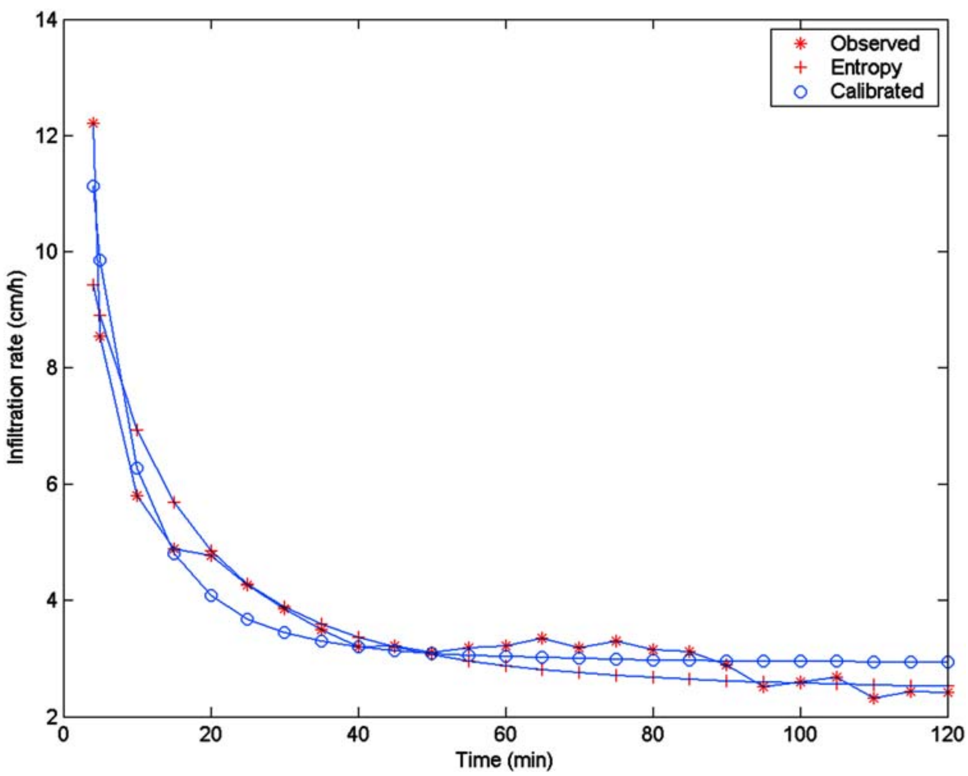

Figure 9. Comparison of infiltration rates computed using the Holtan equation with parameters determined using entropy theory and by calibration with observed infiltration rates for data set I.

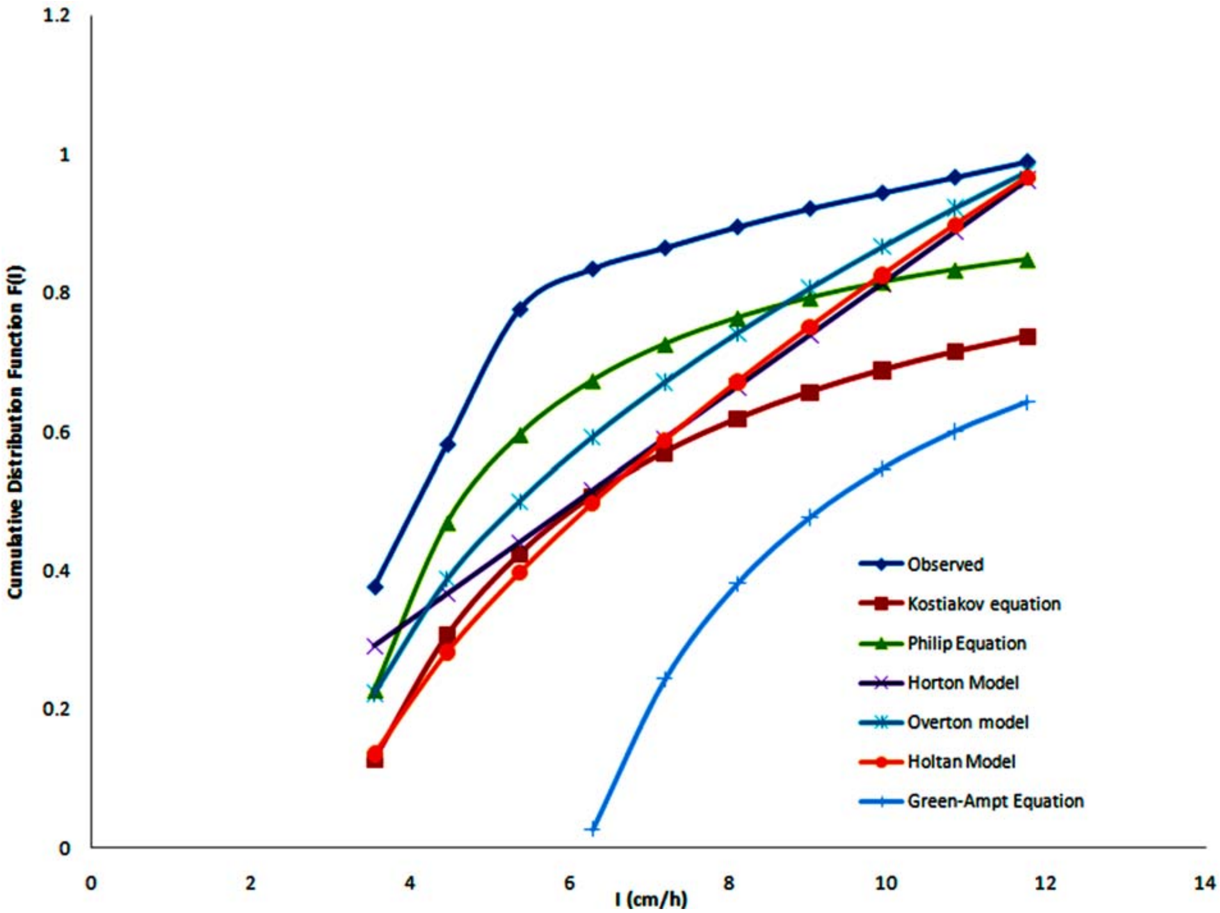

Figure 10. Cumulative distribution functions of different infiltration equations for data set 1.

4.9. Probability Distributions and Entropy of Infiltration Equations

[41] For all six infiltrations CDFs and PDFs were determined both empirically and from the entropy theory (i.e., theoretically) for all four data sets. These CDFs and PDFs were similar in shape and for a sample data set I, these are shown in Figures 10 and 11. The theoretical CDFs and PDFs did not match the empirical CDFs and CDFs. The lack of agreement is not the weakness of the theory because it is due to the weakness of the equations themselves. All that the theory does is uncover the probability distributions underlying these equations. Another reason may be that infiltration observations are not entirely independently random. Furthermore, PDFs and CDFs of different equations are quite different from each other, reflecting the differences in the assumptions and hypotheses of these equations.

[42] Entropy values of all six infiltration equations were computed and are given in Table 2. The probability density function of the Horton equation is a uniform distribution over the length ($I_0 - I_c$) as shown by equation (12b) and hence the entropy of the Horton equation given by equation (21a) will be maximum over this length. This means that the larger the difference between initial infiltration and the steady infiltration the larger will be the entropy value. The implication is that more observations will be needed to better characterize infiltration for the larger difference. For data set I, the Horton entropy value was 2.28 Napier. The PDF of the Kostiakov equation is given by equation (A18), and its entropy by equation (A19). The entropy and hence uncertainty increases with increasing value of steady infiltration rate. For increasing value of steady infiltration rate, more observations will be needed to characterize infiltration. For data set I, the Kostiakov entropy value is 2.88 Napier. The PDF of the Philip equation is given by equation (B11) and its entropy by equation (B16). The PDF has the same shape as that of the Kostiakov equation. The entropy of the Philip equation is 2.19 Napier. The probability density function of the G-A equation is given by equation (C10) and its entropy is given by equation (C17). For data set I, the entropy value of the G-A equation is 2.88, the same as for the Kostiakov equation. The PDF of the Overton equation is given by equation (D10) and its entropy by equation (D17). For data set I, entropy is 1.90 Napier. The PDF of the Holtan equation is given by equation (E4) and its entropy by equation (E18). The Holtan entropy value for data set I is 2.11 Napier. The lowest entropy value was for the Overton equation and the highest entropy value was for Kostiakov and G-A equations. The Holtan, Philip, Horton were in between.

4.10. Comparison of Infiltration Equations

[43] The infiltration equations with parameters estimated using the entropy theory were compared for all four data sets. The Green-Ampt equation deviated more from observations

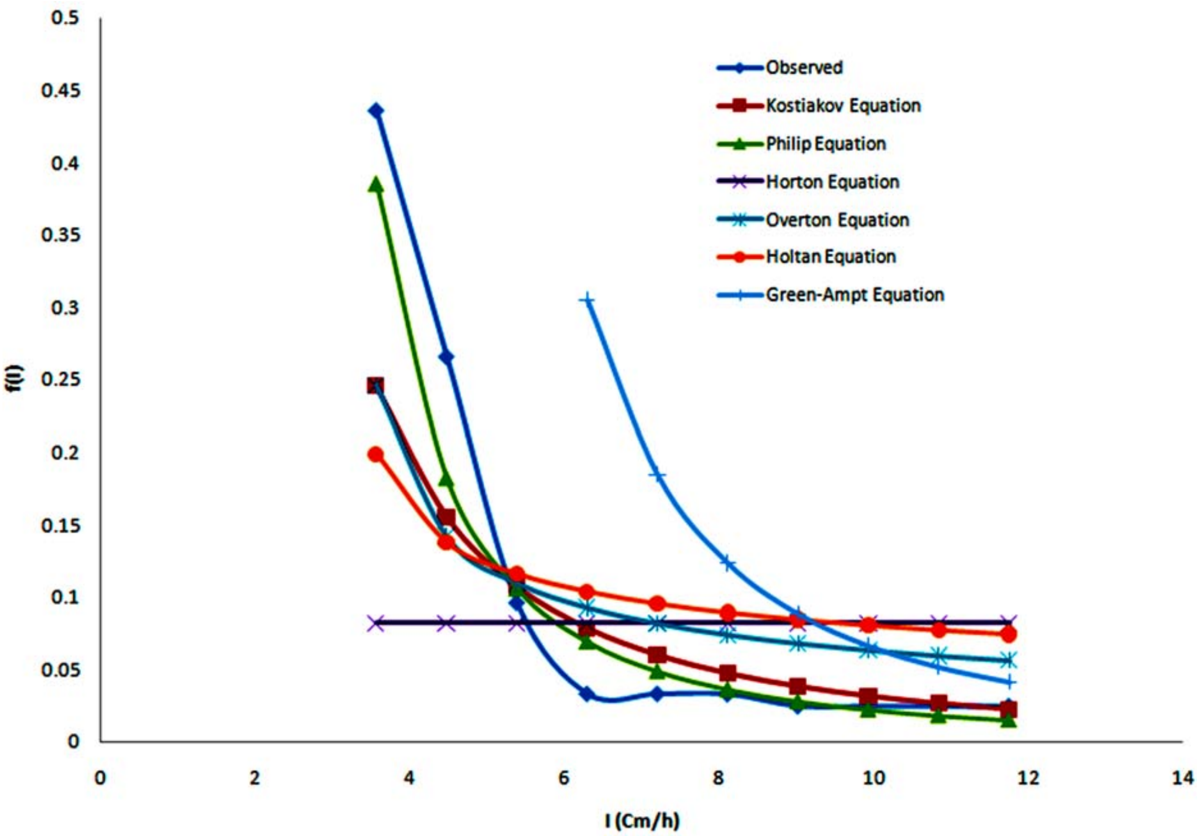

Figure 11. Probability density functions of different infiltration models for data set I.

than did the other equations. The G-A equation had the highest entropy but so had the Kostiakov equation which matched observed infiltration rates quite well. The lowest entropy values were for the Overton equation, followed by the Holtan equation, followed by the Philip equation, followed by the Horton equation. However, because of the differences in the integration limits (or domains of solution) it was difficult to employ the entropy values for selecting the best equation for given sets of data.

4.11. Computation of S From SCS-CN Method

[44] For the data sets employed in this study, an attempt was made to get an estimate of parameter S from the SCS-CN method. For data set I, 70% of the area is covered by weeds and 30% is bare. The soil can be classified in hydrologic group soil B. Referring to the SCS-CN tables, the curve number (CN) for bare soil 86 and the curve number with weeds can be taken as equivalent to crop residue cover which would be 85. The weighted curve number then would be 85.3. Therefore, the value of S would be: 1.72 inches (4.37 cm), which is remarkably close to what (S_1 = 4.17 cm) was determined empirically. The assumption here is that the antecedent moisture condition (AMC) is II. For data II, the weighted curve number would be the same as for data set I as the cover conditions are the same, but the antecedent moisture condition in this case will change from II to III,

Table 2. Entropy Values of Different Infiltration Equations for Data Set I

Equation	Constraints	H	Entropy[a]	I_c	Limits for Integration
Horton	$\int_{I_c}^{I_0} f(I)dI = 1$	$H = \ln[I_0 - I_c]$	2.28	2.42	(I_c, I_0)
Kostiakov	$\int_{I_c}^{I_0} \ln I f(I)dI = \overline{\ln I},\ I_0 \to \infty$	$H = 2 + \ln I_c$	2.88	2.42	(I_c, ∞)
Philip	$\int_{a}^{\infty} \ln I f(I)dI = \overline{\ln I},\ a > 0$	$H = 2 + \ln a_0$	2.19	2.42	(a_0, ∞), $a_0 = 0.5I_c$
Green-Ampt	$\int_{c}^{\infty} \ln(I - I_c)f(I)dI = \overline{\ln(I - I_c)},\ c \to 0$	$H = 2 + \ln I_c$	2.88	2.42	$(2I_c, \infty)$
Overton	$\int_{I_c}^{I_0} \ln(I - I_c)f(I)dI = \overline{\ln(I - I_c)}$	$H = -1 - \ln 0.5 + \ln(I_0 - I_c)$	1.90	3.10	(I_c, I_0)
Holtan	$\int_{I_c}^{I_0} \ln(I - I_c)f(I)dI = \overline{\ln(I - I_c)}$	$H = -\frac{1}{2} - \ln 2/3 + \ln(I_0 - I_c)$	2.11		(I_c, I_0)

[a]Entropy is given in Napiers.

since soil is much wetter and as a result the amount of infiltration is significantly less. Thus, the curve cumber of 85.3 for AMC II would change to 94 for AMC III. This would correspond to the S value of 0.64 inches (1.63 cm), which is significantly higher than that (0.76 cm) obtained empirically. For data set III, 50% of the area is weeds and 50% is bare. It can be considered as a bare soil and therefore the curve number value would be 86 under AMC II. Translating it to AMC III one would obtain a CN value of 94. This would then yield an S value of 0.64 inches (1.63 cm) which is in good agreement with the value (1.68 cm) obtained empirically. For data set IV, the cover condition is the same as for data set III. The curve number would be the same but the AMC II would change to AMC I. Translating the CN value of 86 for AMC II to that for AMC I, one would get a CN value of 72. This would result in a value of S as 1.38 inches (3.51 cm) which is higher than that (2.59 cm) observed empirically. It should be noted that there is not enough information on soil condition, antecedent condition, and antecedent precipitation for the data sets used in order to be able to make an accurate estimate of *S*. This topic needs further investigation which is beyond the scope of this study. For lack of information, it was not possible to compute the S values for other equations and do the comparison.

5. Conclusions

[45] The following conclusions are drawn from this study:

[46] 1. There are three fundamental parameters, including initial infiltration rate (I_0), steady state or constant infiltration rate (I_c), and the maximum soil moisture retention (S) that arise when deriving infiltration equations of Horton, Kostiakov, Philip, Green and Ampt, Overton, and Holtan. Parameters arising in these equations can be expressed in terms of these fundamental quantities which all can be obtained from observations. In this manner the entropy theory renders these infiltration equations nonparametric or parameter-free. In the case of the Overton equation there is a time parameter which indicates the time at which infiltration rate becomes constant. This parameter must be obtained either from observations or by calibration and entropy theory provides no formulation for this time parameter.

[47] 2. The infiltration rates computed by the six equations using entropy theory based parameters compare reasonably well with those computed using parameters obtained by calibration. In the case of the entropy theory parameters are obtained from observations and no calibration is needed.

[48] 3. The entropy theory provides a physical interpretation of infiltration equation parameters.

Appendix A: Kostiakov Equation

[49] Let the constraints be defined as equation (2) and

$$\int_{I_c}^{I_0} \ln I f(I) dI = \overline{\ln I} \tag{A1}$$

where I_c is some small value equal to steady infiltration but tending to 0 and I_0 tending to ∞. Using POME and the method of Lagrange multipliers [*Singh*, 1998], $f(I)$ is obtained as

$$f(I) = \exp[-\lambda_0 - \lambda_1 \ln I] = I^{-\lambda_1} \exp(-\lambda_0) \tag{A2}$$

Equation (A2) contains two Lagrange multipliers λ_0 and λ_1 which can be determined as follows.

[50] Substituting equation (A2) in equation (2), one gets

$$\int_{I_c}^{\infty} I^{-\lambda_1} \exp(-\lambda_0) dI = 1 \Rightarrow \exp(\lambda_0) = -\frac{-\lambda_1 + 1}{I_c^{-\lambda_1+1}} \tag{A3}$$

Inserting equation (A3) in equation (A2), one obtains

$$f(I) = -\frac{-\lambda_1 + 1}{I_c^{-\lambda_1+1}} I^{-\lambda_1} \tag{A4}$$

Equation (A4) contains the Lagrange multiplier λ_1 that is determined using the constraint equation (A1). To that end, equation (A3) can be written as

$$\lambda_0 = -\ln(\lambda_1 - 1) + (1 - \lambda_1) \ln I_c \tag{A5}$$

Differentiating equation (A5) with respect to λ_1, one gets

$$\frac{\partial \lambda_0}{\partial \lambda_1} = \frac{1}{1 - \lambda_1} - \ln I_c \tag{A6}$$

[51] On the other hand, equation (A2) can also be written as

$$\lambda_0 = \ln \int_{I_c}^{\infty} I^{-\lambda_1} dI \tag{A7}$$

Differentiating equation (A7) with respect to λ_1 one obtains

$$\frac{\partial \lambda_0}{\partial \lambda_1} = -\frac{\int_{I_c}^{\infty} I^{-\lambda_1} \exp(-\lambda_0) \ln I dI}{\int_{I_c}^{\infty} \exp(-\lambda_0) I^{-\lambda_1} dI} = -\overline{\ln I} \tag{A8}$$

Equating equations (A6) and (A8), one obtains

$$\lambda_1 = 1 + \frac{1}{\overline{\ln I} - \ln I_c} \tag{A9}$$

Thus, the Lagrange multiplier λ_1 is expressed in terms of the steady infiltration rate and the average of logarithmic infiltration rate.

[52] Substituting equation (A9) in equation (A4), one obtains the probability density function of infiltration rate as

$$f(I) = \frac{1}{\left[\overline{\ln I} - \ln I_c\right]} \frac{I_c^{\frac{1}{\overline{\ln I} - \ln I_c}}}{I^{1 + \frac{1}{\overline{\ln I} - \ln I_c}}} \tag{A10}$$

With the use of equations (A10) and (9c), one can derive a general form of the Kostiakov equation. However, the objective is to derive the Kostiakov equation which results if $\lambda_1 = 2$ or the quantity $1 + [1/(\overline{\ln I} - \ln I_c)]$ is approximated as 2.

[53] Combining equation (A4) or equation (A10) with equation (9c), the result with limits on I from I to ∞ and on J from J to 0 is

$$\frac{I_c S}{I} = J \tag{A11}$$

Recalling that $I = dJ/dt$, equation (A5) can be expressed as

$$\frac{dJ}{dt} = \frac{I_c S}{J} \tag{A12}$$

Integration of equation (A12) yields

$$J = (2I_c S)^{0.5} t^{0.5} \tag{A13}$$

Differentiating equation (A13), one obtains the rate of infiltration:

$$I = \frac{1}{2}(2I_c S)^{0.5} t^{-0.5} \tag{A14}$$

Equation (A13) can be recast as

$$J = at^{0.5} \tag{A15}$$

and equation (A15) as

$$I(t) = 0.5at^{-0.5} \tag{A16}$$

which is the Kostiakov equation with a as parameter expressed as:

$$a = (2I_c S)^{0.5} \tag{A17}$$

[54] The probability density function of the Kostiakov equation can be expressed as

$$f(I) = \frac{I_c}{I^2} \tag{A18}$$

Substituting equation (A18) in equation (1b), the entropy of the Kostiakov equation can be written as

$$H = 2 + \ln I_c \tag{A19}$$

Appendix B: Philip Two-Term Equation

[55] It is assumed that the infiltration rate varies from some constant value $a_0 > 0$ to infinity. The value of constant a_0 is very small. Let the infiltration rate be defined by scaling as $i = I - a_0$. Let there be another constant c which is greater than a_0, perhaps close to some fraction of the steady infiltration rate. Then, the constraints can be defined by equation (2) with limits as c to ∞, and

$$\int_c^\infty \ln i f(i) di = \overline{\ln i} \tag{B1}$$

Using POME and the method of Lagrange multipliers, $f(i)$ is obtained as

$$f(I) = \exp[-\lambda_0 - \lambda_1 \ln i] = i^{-\lambda_1} \exp(-\lambda_0) \tag{B2}$$

Equation (B2) contains two Lagrange multipliers λ_0 and λ_1 which can be determined as follows.

[56] Substituting equation (B2) in equation (2), one gets

$$i = 1 \Rightarrow \exp(\lambda_0) = -\frac{a_0^{-\lambda_1+1}}{\lambda_1 - 1} \tag{B3}$$

Inserting equation (B3) in equation (B2) one obtains

$$f(i) = f(I) = -\frac{-\lambda_1 + 1}{a_0^{-\lambda_1+1}} i^{-\lambda_1} \tag{B4}$$

Equation (B4) contains the Lagrange multiplier λ_1 which can be determined using the constraint equation (B1). To that end, equation (B3) can be expressed as

$$\lambda_0 = (1 - \lambda_1) \ln a_0 - \ln(\lambda_1 - 1) \tag{B5}$$

Differentiating equation (B5), one gets

$$\frac{\partial \lambda_0}{\partial \lambda_1} = -\frac{1}{\lambda_1 - 1} - \ln c \tag{B6}$$

[57] On the other hand, equation (B3) can be written as

$$\lambda_0 = \ln \int_c^\infty i^{\lambda_1} di \tag{B7}$$

Differentiating equation (B7), the result is

$$\frac{\partial \lambda_0}{\partial \lambda_1} = -\frac{\int_c^\infty i^{-\lambda_1} \exp(-\lambda_0) \ln i di}{\int_c^\infty i^{-\lambda_1} \exp(-\lambda_0) di} = \overline{\ln i} \tag{B8}$$

Equating equations (B6) and (B8), one gets

$$\lambda_1 = 1 + \frac{1}{\overline{\ln i} - \ln c} \tag{B9}$$

The probability density function of infiltration rate is now expressed as

$$f(i) = f(I) = \frac{1}{\left[\overline{\ln i} - \ln c\right]} \frac{a^{\frac{1}{\overline{\ln i} - \ln c}}}{i^{1 + \frac{1}{\overline{\ln i} - \ln c}}} \tag{B10}$$

Using equation (B10) and equation (9c), one obtains a general form of the Philip equation. However, the objective is to derive the Philip equation which results if $\lambda_1 = 2$ or $1 + 1/[\overline{\ln i} - \ln c]$ is approximated as 2. Thus, taking $\lambda_1 = 2$, one obtains

$$f(i) = \frac{c}{i^2}; \qquad f(I) = \frac{c}{(I - c)^2} \tag{B11}$$

Equation (B11) is the probability density function of the Philip equation. Combining equation (B11) with equation (9c), the result with limits on i from i to ∞ and on J from J to 0 is

$$\frac{cS}{i} = J \tag{B12}$$

Recalling that $i = dJ/dt$, equation (B12) can be expressed as

$$\frac{dJ}{dt} = \frac{cS}{J} \Rightarrow J = (2cS)^{0.5} t^{0.5} \tag{B13}$$

Differentiating equation (B13), one obtains the rate of infiltration:

$$i = \frac{1}{2}(2cS)^{0.5} t^{-0.5} \tag{B14}$$

Equation (B14) can be written in terms of the original infiltration rate I as

$$I(t) = a_0 + 0.5(2cS)^{0.5} t^{-0.5} = a_0 + bt^{-0.5}, \quad b = 0.5(2cS)^{0.5} \tag{B15a}$$

which is the Philip two-term equation. For practical purposes c and a_0 can be considered equivalent and hence equation (B15a) can be written as

$$I(t) = a_0 + 0.5(2a_0S)^{0.5} t^{-0.5} = a_0 + bt^{-0.5}, \quad b = 0.5(2a_0S)^{0.5} \tag{B15b}$$

Using equation (B11) in equation (1b) one obtains the entropy of the Philip equation:

$$H = 2 + \ln c \approx 2 + \ln a_0 \tag{B16}$$

Appendix C: Green-Ampt Equation

[58] Let the constraints be defined by equation (2) with limits as b_0 to c_0 where b_0 would tend to ∞, and c_0 to 0, and

$$\int_{c_0}^{\infty} \ln(I - I_c) f(I) dI = \overline{\ln(I - I_c)} \tag{C1}$$

Using POME and the method of Lagrange multipliers, $f(I)$ is obtained as

$$f(I) = \exp[-\lambda_0 - \lambda_1 \ln(I - I_c)] = (I - I_c)^{-\lambda_1} \exp(-\lambda_0) \tag{C2}$$

Equation (C2) contains two Lagrange multipliers λ_0 and λ_1 which can be determined as follows. Substituting equation (C2) in equation (2), one gets

$$\int_{c_0}^{b_0} (I - I_c)^{-\lambda_1} \exp(-\lambda_0) dI = 1 \Rightarrow \exp(\lambda_0) = \frac{(b_0 - I_c)^{-\lambda_1+1}}{-\lambda_1 + 1} - \frac{(c_0 - I_c)^{-\lambda_1+1}}{-\lambda_1 + 1} \tag{C3}$$

Inserting equation (C3) in equation (C2) one obtains

$$f(I) = \frac{(-\lambda_1 + 1)(I - I_c)^{-\lambda_1}}{\left[(b_0 - I_c)^{-\lambda_1+1} - (c_0 - I_c)^{-\lambda_1+1}\right]} \tag{C4}$$

Equation (C4) contains the Lagrange multiplier λ_1 which can be determined using the constraint equation (C1). To that end, equation (C3) can be written with $b_0 \rightarrow \infty$, $c_0 \rightarrow 0$ as

$$\lambda_0 = -\ln(\lambda_1 - 1) + (1 - \lambda_1)\ln(-I_c) \tag{C5}$$

Differentiating equation (C5) with respect to λ_1, one gets

$$\frac{\partial \lambda_0}{\partial \lambda_1} = -\frac{1}{\lambda_1 - 1} - \ln(-I_c) \tag{C6}$$

On the other hand, equation (C3) can also be written as

$$\lambda_0 = \ln \int_{c_0}^{\infty} (I - I_c)^{-\lambda_1} dI \tag{C7}$$

Differentiation of equation (C7) leads to

$$\frac{\partial \lambda_0}{\partial \lambda_1} - -\overline{\ln(I - I_c)} \tag{C8}$$

Equating equations (C6) and (C8), one gets an expression for λ_1:

$$\lambda_1 = 1 + \frac{1}{\overline{\ln(I - I_c)} - \ln(-I_c)} \tag{C9a}$$

It should be noted that the term $\ln(-I_c)$ in equation (C9a) is undefined for any finite value of I_c. If I_c is allowed to tend to zero then $(-\ln(-I_c))$ goes to zero and equation (C9a) can be approximated as

$$\lambda_1 = 1 + \frac{1}{\overline{\ln(I - I_c)}} \tag{C9b}$$

[59] Thus, the probability density function of the infiltration rate can be expressed by equation (C4) with λ_1 given by equation (C9a) or (C9b). However, the objective is to derive the Green-Ampt equation which results if $\lambda_1 = 2$. Therefore, equation (C4) becomes

$$f(I) = \frac{I_c}{(I - I_c)^2} \tag{C10}$$

Equation (C10) is the probability density function of the Green-Ampt equation. It should however be noted that this density function is valid only for $2I_c \leq I < \infty$, not for the entire first quadrant.

[60] Combining equation (C10) with equation (9c), the result is

$$\frac{I_c dI}{(I - I_c)^2} = -\frac{1}{S} dJ \tag{C11}$$

Integrating with limits for I from ∞ to I and for J from 0 to J,

$$\frac{I_c}{(I - I_c)} = -\frac{J}{S} \tag{C12}$$

Recalling that $I = dJ/dt$, equation (C12) can be expressed as

$$\frac{dJ}{dt} = \frac{SI_c}{J} + I_c \tag{C13}$$

Solution of equation (C13), with the condition that $t = 0$, $J = 0$, is

$$t = \frac{1}{I_c}\left[J - S\log\left(1 + \frac{J}{S}\right)\right] \tag{C14}$$

Equation (C14) can be expressed as

$$t = \frac{1}{I_c}\left[J - \frac{a_1}{I_c}\log\left(1 + \frac{J}{a_1/I_c}\right)\right] \tag{C15}$$

where

$$a_1 = SI_c \tag{C16}$$

Equation (C16) is the Green-Ampt equation in which parameter I_c can be interpreted as equal to the saturated hydraulic conductivity and parameter S equal to the product of the capillary suction at the wetting front and the initial moisture deficit.

[61] Using equations (1b) and (C10), the entropy of the Green-Ampt equation can be written as

$$H = 2 + \ln I_c \tag{C17}$$

Appendix D: Overton Model

[62] Let the constraints be defined by equation (2) and

$$\int_{I_c}^{I_0} \ln(I - I_c) f(I) dI = \overline{\ln(I - I_c)} \tag{D1}$$

Using POME and the method of Lagrange multipliers, $f(I)$ is obtained as

$$f(I) = \exp[-\lambda_0 - \lambda_1 \ln(I - I_c)] = (I - I_c)^{-\lambda_1}\exp(-\lambda_0) \tag{D2}$$

Equation (D2) contains two Lagrange multipliers λ_0 and λ_1 which can be determined as follows.

[63] Substituting equation (D2) in equation (2), one gets

$$\int_{I_c}^{I_0} (I - I_c)^{-\lambda_1}\exp(-\lambda_0) dI = 1 \Rightarrow \exp(\lambda_0) = \frac{(I_0 - I_c)^{-\lambda_1 + 1}}{-\lambda_1 + 1} \tag{D3}$$

Inserting equation (D3) in equation (D2)

$$f(I) = \frac{(-\lambda_1 + 1)(I - I_c)^{-\lambda_1}}{(I_0 - I_c)^{-\lambda_1 + 1}} \tag{D4}$$

Equation (D4) contains the Lagrange multiplier λ_1 which can be determined with the use of the constraint equation (D1). To that end, equation (D3) can be written as

$$\lambda_0 = (-\lambda_1 + 1)\ln(I_0 - I_c) - \ln(-\lambda_1 + 1) \tag{D5}$$

Differentiating equation (D5), one gets

$$\frac{\partial \lambda_0}{\partial \lambda_1} = -\ln(I_0 - I_c) + \frac{1}{-\lambda_1 + 1} \tag{D6}$$

On the other hand, equation (D3) can be written as

$$\lambda_0 = \ln \int_{I_c}^{I_0} (I - I_c)^{-\lambda_1} dI \tag{D7}$$

Differentiating equation (D7) and recalling equation (D1), one gets

$$\frac{\partial \lambda_0}{\partial \lambda_1} = -\overline{\ln(I - I_c)} \tag{D8}$$

Equating equations (D6) and (D8), the result is

$$\lambda_1 = 1 - \frac{1}{\ln(I_0 - I_c) - \overline{\ln(I - I_c)}} \tag{D9}$$

Substitution of equation (D9) in equation (D4) yields the general expression of the PDF and with the use of equation (9c) one obtains the general form of the Overton equation. The objective however is to derive the Overton equation which results if $\lambda_1 = 0.5$ or $1 - \{1/[\ln(I_0 - I_c) - \overline{\ln(I - I_c)}]\}$ is about 0.5. Then equation (D4) becomes

$$f(I) = \frac{0.5(I - I_c)^{-0.5}}{(I_0 - I_c)^{0.5}} \tag{D10}$$

Equation (D10) is the probability density function of the Overton model.

[64] Substituting equation (D10) in equation (9c), one obtains

$$-\frac{1}{S}dJ = \frac{0.5(I - I_c)^{-0.5}}{(I_0 - I_c)^{0.5}} dI \tag{D11}$$

Integration of equation (D11) yields

$$I = \frac{(I_0 - I_c)}{S^2}J^2 + I_c \tag{D12}$$

Recalling equation (8), equation (D12) with limits on t from t to t_c and on J from J to J_c (constant) gives

$$J = J_c - S\sqrt{\frac{I_c}{(I_0 - I_c)}}\tan\left[\frac{\sqrt{I_c(I_0 - I_c)}}{S}(t_c - t)\right] \tag{D13}$$

Differentiating equation (D13) leads to

$$I(t) = I_c \sec^2\left[\frac{\sqrt{I_c(I_0 - I_c)}}{S}(t_c - t)\right] \tag{D14}$$

Let

$$(I_0 - I_c) = a_2 S^2 \tag{D15}$$

Equation (D14) becomes

$$I(t) = I_c \sec^2\left[\sqrt{a_2 I_c}(t_c - t)\right] \tag{D16}$$

Equation (D16) is the Overton model.

[65] Using equation (D5) in equation (1b), one obtains the entropy of the Overton equation:

$$H(I) = -1 - \ln 0.5 + \ln(I_0 - I_c) \tag{D17}$$

Appendix E: Holtan Model

[66] Analogous to the Horton equation, let i define the excess infiltration rate $(I - I_c)$ varying from 0 to i_0 where $i_0 = I_0 - I_c$. Then, the constraints can be defined by equation (2) and equation (D1) (with proper infiltration rate in mind). Using POME and the method of Lagrange multipliers, $f(i)$ is obtained as equation (D2) and eventually equation (D4):

$$f(I) = \frac{(-\lambda_1 + 1)i^{-\lambda_1}}{(i_0)^{-\lambda_1+1}} = b_0 i^{-\lambda_1} \tag{E1}$$

where

$$b_0 = \frac{(-\lambda_1 + 1)}{(i_0)^{-\lambda_1+1}} \tag{E2}$$

Let

$$-\lambda_1 = \frac{1 - m}{m} \tag{E3}$$

Equation (E1) can be recast as

$$f(i) = b_0 i^{\frac{1-m}{m}} \tag{E4}$$

and equation (E2) as

$$b_0 = \frac{1}{m(i_0)^{1/m}} \tag{E5}$$

Equation (E4) is the probability density function of the Holtan equation. Equation (E4) contains the Lagrange multiplier λ_1 which can be determined using the constraint equation (D1). To that end, substitution of equation (E1) in equation (2) yields

$$\exp(\lambda_0) = \frac{(i_0)^{-\lambda_1+1}}{-\lambda_1 + 1} \tag{E6}$$

which can be expressed as

$$\lambda_0 = (-\lambda_1 + 1)\ln i_0 - \ln(-\lambda_1 + 1) \tag{E7}$$

Differentiating equation (E7) with respect to λ_1 one gets

$$\frac{\partial\lambda_0}{\partial\lambda_1} = -\ln i_0 + \frac{1}{-\lambda_1 + 1} \tag{E8}$$

Equation (E8) can be shown to equal $-\overline{\ln i}$. Therefore, equation (E8) can be written as

$$\lambda_1 = 1 - \frac{1}{\ln i_0 - \overline{\ln i}} \tag{E9}$$

Recalling the definition of λ_1, one obtains

$$m = \ln i_0 - \overline{\ln i} = \ln(I_0 - I_c) - \overline{\ln(I - I_c)} \tag{E10}$$

Thus, m can also be expressed in terms of physically measurable quantities. Thus, the probability density function of the infiltration rate is given by equation (E4). For the Holtan equation, parameter m through simulation, was found to be approximately 2.3.

[67] Substituting equation (E1) in equation (9c), one obtains

$$dJ = -Sb_0 i^{\frac{1-m}{m}} di \tag{E11}$$

Integration of equation (E11) yields

$$S - J = Sb_0 m\, i^{\frac{1}{m}} \tag{E12}$$

Equation (E12) can be expressed as

$$(mb_0 S)^m (S - J)^{-m}\, dJ = dt \tag{E13}$$

Integrating equation (E13), one obtains

$$J = S - \left[S^{1-m} - \frac{(1-m)}{(Sb_0 m)^m} t\right]^{\frac{1}{1-m}} \tag{E14}$$

Differentiation of equation (E14) with respect to t and simplification yield

$$i = a_3\left[S^{1-m} - a(1-m)t\right]^{\frac{m}{1-m}} \tag{E15}$$

where

$$a_3 = \frac{i_0}{S^m} \tag{E16}$$

Equation (E15) can be written in original terms as

$$I = I_c + a_3[S^{1-m} - a_3(1-m)t]^{\frac{m}{1-m}}, \quad a_3 = \frac{(I_0 - I_c)}{S^m} \tag{E17}$$

Equation (E17) is the Holtan equation with parameter a_3 given by equation (E16). The Holtan equation has two parameters, a_3 and m, both of which are expressed in terms of physically measurable quantities.

[68] Substituting equation (E4) in equation (1b), the entropy of the Holtan model can be expressed as

$$\begin{aligned} H(I) = & -\frac{1}{m}\int_{I_c}^{I_0} \frac{(I - I_c)^{\frac{1-m}{m}}}{(I_0 - I_c)^{\frac{1}{m}}} \ln \frac{(I - I_c)^{-\frac{1-m}{m}}}{(I_0 - I_c)^{\frac{1}{m}}} dI = -\frac{1}{2} \\ & - \ln 2/3 + \ln(I_0 - I_c) \end{aligned} \tag{E18}$$

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